International Institute of Information Technology, Hyderabad
MA4.101-Real Analysis (Monsoon-2026)
Quiz 2

Time: 60 Minutes Total Marks: 20

Question 1 [10 Marks]. Let $x_n \to 0$. Let (a_n) be the sequence of arithmetic means, defined by $a_n = \frac{x_1+x_2+....x_n}{n}$. Then show that $a_n \to 0$.

Question 2 [10 Marks]. Define the sequence $(a_n)_{n\geq 1}$ by $a_1 = 4$ and

$$a_n = \begin{cases} -1 + \frac{1}{k}, & \text{if } n = k! \text{ for some integer } k \geq 2, \\ 1 - \frac{1}{n}, & \text{otherwise.} \end{cases}$$

Here $k! = k \times (k-1) \times \cdots \times 2 \times 1$ for $k \geq 2$. Answer the following questions.

(a) **[1 Mark]** Write out the first six terms $a_1, \ldots, a_6$.

(b) **[2 Marks]** Show that the sequence $(a_n)_{n\geq 1}$ is both bounded from below and bounded from above.

(c) **[3 Marks]** Compute $\sup(a_n)_{n\geq 1}$ and $\inf(a_n)_{n\geq 1}$.

(d) **[4 Marks]** Compute $\limsup_{n\to\infty} a_n$ and $\liminf_{n\to\infty} a_n$.